

Answer all of the following questions. Calculators, Pagers and mobile telephones are NOT allowed.

1. Let  $f(x) = \frac{9^x - 1}{9^x + 1}$ ,  $-\infty < x < +\infty$ .

Show that  $f^{-1}$  exists and state its domain and range and compute  $f^{-1}(x)$ . (4 points)

2. (a) Find  $\frac{dy}{dx}$  if  $y = \frac{(1+x^2)^{\tan^{-1}(e^x)}}{(e^{\sin x})\sqrt[3]{1+x^3}}$  (4 points)

(b) Find  $\lim_{x \rightarrow 0^+} \left(\frac{3^x + 5^x}{2}\right)^{\frac{1}{x}}$ , if it exists. (4 points)

3. Evaluate the following integrals (4 points each)

(a)  $\int \tanh x \operatorname{sech}^4 x dx$

(b)  $\int (1 + \cos x)^{\frac{3}{2}} dx$

(c)  $\int \frac{(x-5)^2}{\sqrt{-x^2 + 10x - 24}} dx$

(d)  $\int \frac{1}{x^2} \tan^{-1}\left(\frac{1}{x}\right) dx$

(e)  $\int \frac{e^{-x}}{e^x - 1} dx$

4. Evaluate the improper integral  $\int_1^{+\infty} \frac{\ln x}{x^2} dx$  if it converges. (4 points)

5. Sketch in polar coordinates the cardioid  $r = 1 + \cos \theta$  and the circle  $r = \sin \theta$  and find the area of the region outside the cardioid and inside the circle. (4 points)

6. Show that the graph of the equation  $x^2 - 3y^2 + 2x + 6y + 1 = 0$  is a hyperbola. Determine the center, vertices, foci and sketch the graph. (4 points)

7. (a) Find the equation of the sphere whose center is the midpoint between  $P(3, 4, 5)$  and  $Q(-1, 2, -3)$  and contains the point  $(5, 2, 2)$ . (3 points)

(b) Find the area of the triangle determined by the points  $P(1, 1, 1)$ ,  $Q(0, 3, 0)$  and  $R(2, 3, 1)$ . (3 points)

Good Luck